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D. Skrzyniowska

Cracow University of Technology

K. Maczek

Cracow University of Technology

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A MODEL OF AIR OUTFLOWING FROM A LUBRICATED POSITIVE DISPLACEMENT COMPRESSOR

DOROTA SKRZYNIOWSKA, KAZIMIERZ MACZEK
FACULTY OF ENVIRONMENTAL ENGINEERING, CRACOW UNIVERSITY OF TECHNOLOGY
Warszawska Street 24, 31-155 KRAKOW, POLAND

ABSTRACT

This paper deals with description of the real mixture of moist air and lubricant (at elevated pressure and temperatures) which outflow from a positive displacement compressor and the main point of this paper is to describe thermodynamic formulas for calculations. Compressed air is very important medium used in industry (e. g. shops, pneumatics, transport, pharmaceutical industry, etc.). It is obtained during air compression mainly in a positive displacement compressor. The cylinder – piston assembly, the reciprocating movement and friction of piston give the main characteristic of such compressors. To reduce the friction lubricant is used. Until now in thermodynamic models and calculations air is not considered mixture with lubricant but as an ideal pure gas. In this paper authors describe mixture consisting of moist air and lubricant by means of differential equations mass conservation law, momentum conservation and energy conservation law.

NOMENCLATURE

\bar{a} – factor [-],
 a_{pist} – acceleration of piston [m/s^2],
 A_p – cross sectional area of cylinder [m^2],
 B – second virial coefficient of mixture [-],
 $B(t)$, $C(t)$, $D(t)$ – second, third, fourth virial coefficient [-],
 B_{gg} – 2. virial coefficient of air [-],
 B_{wg} – 2. interaction coefficient of water vapour-air [-],
 B_{ww} – 2 virial coefficient of water vapour [-],
 c_l – the specific heat under the constant pressure for the lubricant [$J/kg K$],
 c_s – relative clearance volume [-],
 c_{pg} – specific heat of dry air at constant pressure [$J/kg K$],
 c_{pw} – specific heat of water vapour at constant [$J/kg K$],
 c_{pll} – specific heat of liquid lubricant [$J/kg K$],
 c_{pvl} – specific heat of vapour lubricant [$J/kg K$],
 c_m – the specific heat under the constant pressure for the mixture [$J/kg K$],
 D_e – equivalent diameter [m],
 D_p – diameter of piston [m],
 D_{in} – inside diameter of cylinder [m],
 D_{out} – outside diameter of cylinder [m],
 f – Fanning friction factor [-],
 g – gravitational acceleration [m/s^2],
 h – specific enthalpy [J/kg],
 m – mass [kg],
 M – partial mass [$kg/kmol$],
 Ob – cross-sectional circumference of cylinder [m],
 p , p_o , p_k , p_r – pressure, inlet pressure, outlet pressure, reduced pressure [Pa],
 \dot{q}_z – rate of heat flow [W/m^2],
 R – gas constant,
 r_o – latent heat of vaporisation of water at $0^\circ C$ [J/kg],
 s – piston stroke [m],
 T , t – temperature [K], [$^\circ C$],
 T_m , T_e , T_r , t_b – temperature of mixture, of ambient air, reduced temperature [K], boiling point [$^\circ C$],

v – molar volume of the mixture [m^3/kg],
 w – velocity [m/s],
 x_i – mole fraction of lubricant in mixture [-],
 x_w – mole fraction of water vapour in moist air [-],
 x_g – mole fraction of air in moist air [-],
 x – mixing ratio [(kg water)/kg dry air)],
 V, V_l, V_c, V_s – volume of mixture, lubricant, cylinder, clearance volume [m^3],
 z – coordinate along the length of cylinder [m],
 α – heat transfer coefficient [$\text{W/m}^2 \text{K}$],
 β – volume fraction [-],
 κ – polytropic exponent [-],
 λ_f – friction factor (of resistance) [-],
 λ_i, λ_m – coefficient of the heat conductivity for i – component, mixture [W/m K],
 μ – coefficient of dynamic viscosity [N s/m^2],
 μ_1 – coefficient of dynamic viscosity of the air [N s/m^2],
 ν – coefficient of kinematic viscosity [m^2/s],
 ξ – coefficient of compressibility [-],
 $\rho, \rho_a, \rho_l, \rho_{15}^{15}$ – density of mixture, air, lubricant [kg/m^3], specific density,
 τ – time [s],
 τ_o – tangential stress [Pa],

INTRODUCTION

The humid air, in thermodynamics, is considered a mixture of ideal gases. However, such an approach is not appropriate enough, when taking into account an influence of higher pressures and temperatures we are dealing in the compressor with.

H. Vendelbo Holm formulates a description of compressed humid air as real mixture. H. Vendelbo Holm uses Mollier's graphs (h and x) describe an influence of different pressures. He argues that there are differences in basic thermodynamic parameters such as an enthalpy of ideal and real mixtures [8]. Carotenuto and M. De Dell'Isola describe simple dependencies for humid air as real mixture. They claim that the real model of humid air is more precise [2]. One needs to emphasize at this point that each analysis relating to humid air with higher pressure, the authors deal with thus humid air as a binary mixture of real gases, water vapour and dry air.

The lubricating substances could be divided into natural ones (mineral oils), synthetic and half-synthetic ones. The mineral oils further on could be divide into: 1/ oils with a small amount of aromatic hydrocarbons, known as paraffin oils; 2/ oils with a majority of aromatic hydrocarbons, known as naphthalene oils; 3/ oils with a majority of aromatic hydrocarbons supplemented by groups of simple or alkyl rings. All these oils were elementary lubrication factors used in technology of air-compression and refrigeration. The progress of technique results in increasing number of lubrication substances being used. The wide scope of research carried out in this field resulted in creation of many new synthetic lubrication substances.

The compressed air class - depending on a form of oil that is a part of this air – has been derived from the ISO – international norm. This norm (Chapter 5.3 „Oil”) states clearly that the air that is compressed in the oil compressors could be supplemented by oil (mineral or synthetic one) in the following forms:

1. liquid, 2. aerosol, 3 vapour

In the cylinder of positive air-compressor where a lubricating substance is used as a sealing, cooling and lubricating, actually is compressed the mixture of humid air and lubricating substance [9]. During the lubricating of cylinder's smooth surface a so-called oil-film is emerging on the cylinder's wall. Further on, a partial dispersion of lubricating substance takes place in the working space of compressor that results in appearance of aerosol. As a consequence of the piston's forward-backward movement and its friction against the cylinder's wall, the wall's temperature is arising. If the temperature is high enough, the wall „dries out”. The oil takes over the heat from the cylinder's wall (cooling function) and turns into vapour. In the compressors' cylinder a structure of flux of the air and lubricating substance mixture in the form of liquid, steam and aerosol is emerging. The heat that is transferred during the exchange between the air and external environment is described as the external heat of transformation \dot{q}_z . The value of this heat and its sign influence a pace of thermodynamic transformation taking place in the compressor directly.

It is assumed in the paper that the process of compression is accompanied of exchange of the heat with the environment. Thus, in order to solve equation systems constituting a mathematics model of compression process what is needed is knowledge of a density of the stream of heat relating to the length of piston stroke.

THE MATHEMATICS MODELING OF A MIXTURE OF MOIST AIR AND LUBRICANT IN THE VERTICAL CYLINDER

The positive displacement compressors are transfer machines for compression and delivery of gases or vapour. In the case of the air positive displacement compressors the air is compressed. It is assumed that the compressor transfers of two-phase mixture, in the vertical pipe, non-slip (phases are moving with the same velocity, acceleration of a stream is caused by external or internal influence is equivalent with the same acceleration of both phases) [5]. The compression that takes place in the compressor is a diabatic one (exchange of the heat with the environment – cooling) Fig. 1 [7].

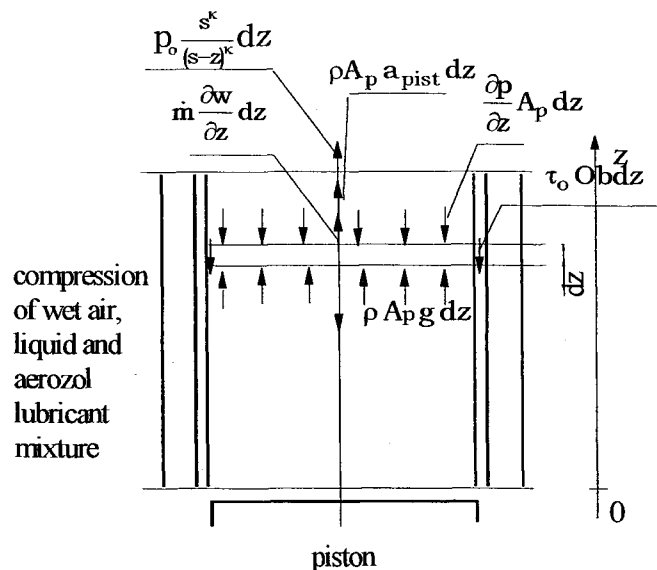


Fig. 1. Forces operative on elementary layer of fluid

All these assumptions and simplifications enable an adoption of the model of homogenic two-phase flux (known also as pseudo-phase one) in the vertical pipe for the description of the air-compression process. The conclusion that could be drawn upon the above outlined characteristic of the compression

process is that the analysed case can be treated as stationary ($\frac{\dot{m}}{A_p} = \text{const}$) and one-dimensional flux of

air mixture with components. The vector of velocity is directed along the cylinder's pipe-axe line. The velocity of flux, enthalpy, specific volume and other values are treated as unambiguous functions of the location of variable z directed along the pipe axe. At the same time the heat of conductivity along the pipe axe is ignored. The thermodynamic features and parameters of the mixture, constants in the cross-section of the pipe vary along the way of flux. These changes are described by equations typical for the mixtures.

During the functioning of the compressor we are dealing with a process of friction and lubrication. The friction is characterised by mechanic losses. This energy is transformed or it is being dispersed. According to Kostecki (Kostecki's theory), almost entire work of the friction is transferred into the heat and it is being dispersed. The part of the heat is carried away from a tribological system in the form of conduction, convection and radiation and finally the part of the heat warms up a lubricating substance and causes its transformation from liquid into vapour.

MASS CONSERVATION LOW, MOMENTUM CONSERVATION, ENERGY CONSERVATION LAW

In the analysed case of compression air-lubricant mixture we are dealing with the haze system. The phenomenological method has been adopted here to describe the model. This method assumes that it is justified to treat two-phase liquid as the two penetrating and interacting continuos units.

The two-phase as well as one-phase flux is governed by the rules mass conservation low, momentum conservation, energy conservation law supplemented by constitutive equations (i.e. equations of state, strain, momentum transport and energy transport as well as chemical features of the liquid). Rules the governing equations of the flow (continuity, momentum and energy) can be displayed either in the form of integral or differential.

As the mass conservation low adapted to the flowing mixture can be used to equation:

$$A_p \left(\frac{\partial \rho}{\partial \tau} \right) + \frac{\partial \dot{m}}{\partial z} = 0 \quad (1)$$

The movement conservation low describes the balance of forces an elementary layer of mixture being in the flow is influenced by:

$$\frac{\partial \dot{m}}{\partial \tau} - A_p \frac{dp}{dz} = \tau_0 * Ob + A_p \rho \frac{wdw}{dz} - A_p \rho g + A_p a_{pist} + p_o \frac{s^\kappa}{(s-z)^\kappa} \quad (2)$$

The energy conservation law (I principal of thermodynamics):

$$\rho A_p \frac{\partial h}{\partial \tau} + \frac{d \left[\dot{m} \left(h + \frac{w^2}{2} \right) \right]}{dz} = \pi D_{in} \dot{q}_z - A_p \rho g w \quad (3)$$

Based upon the above adopted assumptions and reductions the equations (1), (2) and (3) can be put it in the following way:

$$A_p \frac{dp}{dz} = -\tau_0 * Ob - A_p \rho \frac{wdw}{dz} + A_p \rho g + A_p a_{pist} + p_o \frac{s^\kappa}{(s-z)^\kappa} \quad (4)$$

$$\frac{d \left[\dot{m} \left(h + \frac{w^2}{2} \right) \right]}{dz} = \pi D_{in} \dot{q}_z - A_p \rho g w \quad (5)$$

The ultimate forms of the mass conservation law, momentum conservation, energy conservation law are the following:

$$A \frac{dp}{dz} = -0,125 \pi \lambda_f D_{in} \rho w^2 - A_p \frac{wdw}{dz} + A_p g + A_p a_{pist} + p_o \frac{s^\kappa}{(s-z)^\kappa} \quad (6)$$

$$\frac{d \left[\dot{m} \left(h + \frac{w^2}{2} \right) \right]}{dz} = \pi D_{in} \dot{q}_z - A_p \rho g w \quad (7)$$

The friction factor λ_f (also known as f – the Fanning's coefficient of friction) is a function of the Reynolds' coefficient

$$Re = \frac{w D_e}{\nu} \quad (8)$$

for the rough movement when $Re \leq 80000$ the Blasius's equation is also obligatory:

$$\lambda_f = 0,316 (Re)^{-0,25} \quad (9)$$

The value of density of the stream of heat is constant in special cases only, usually is $\dot{q}_z \neq \text{const.}$ Having known the values of T_m and T_e temperatures and value convective heat-transfer coefficient α , the heat flux density can be define in the following way:

$$\dot{q}_z = \alpha [T_m - T_e] \quad (10)$$

The Nusselt's criterion serves as a method of identification of the exchange of heat coefficients. The exchange of heat between the compressed air and film of oil takes place due to convection. According to [3, 4, 7] the Nusselt's coefficient for the compressors revised by Adair and others (1972) is:

$$Nu = 0,053 Re^{0,8} Pr^{0,6} \quad (11)$$

$$Pr = \frac{\nu}{a} \quad (12)$$

where:

$$a = \frac{\lambda_m}{\rho c_m} \quad (13)$$

However,

$$\alpha = \frac{Nu \lambda_m}{D_w} \quad (14)$$

Having put (13) and (14), one received:

$$\dot{q}_z = \frac{\lambda_m}{D_{in}} 0,053 \left(\frac{w D_{in}}{\nu} \right)^{0,8} \left(\frac{\nu}{a} \right)^{0,6} [T_m - T_e] \quad (15)$$

In the two-phase fluxes where we are dealing with mixtures, the identification of density is extremely complex since the density of mixture is not regulated by the rule of additivity [6].

Due to the fact that the structure of the two-phase stream is not known one can use certain median values of dependencies. There are few theoretical formulas adopted for small volume concentrations of the dispersed phase β . The most popular in this context is the D.G. Thomas' formula (1965) adopted for the whole interval of β concentrations [6]:

$$\mu = \mu_1 \left[1 + 2,5\beta + 10,05\beta^2 + 0,00273 \exp(16,6\beta) \right] \quad (16)$$

The kinematics coefficient of density can identify upon the following dependence:

$$v = \frac{\mu}{\rho} \quad (17)$$

The density of the two-phase mixture can be calculated depending on what methods have been adopted to express the proportions of components the mixture consists of:

$$\rho = (1 - \beta)\rho_a + \beta\rho_1 \quad (18)$$

where:

$$\beta = \frac{V_1}{V} \quad (19)$$

The Rajender Prakash' and Rajender Singh' formula of the whole area of surface of the exchange of heat takes a form of:

$$A(z) = \frac{\pi D_{in}^2}{4} + 4 \frac{V_c}{D_{in}} + \pi D_{in} z \quad (20)$$

whereas :

$$V_c = c_s V_s = c_s \frac{\pi D_p^2}{4} s \quad (21)$$

for the compressors one adopts: $c_s = 0,04$

Finally:

$$A = \frac{\pi D_{in}^2}{4} + 0,04 \pi D_{in} s + \pi D_{in} z \quad (22)$$

The perimeter of cross-section (wetted perimeter):

$$Ob = 2\pi \frac{D_{in}}{2} = \pi D_{in} \quad (23)$$

The equivalent diameter equals:

$$D_e = \frac{6\pi \left(\frac{D_{in}}{2} \right)^2 z}{\pi D_{out} + 2\pi \left(\frac{D_{in}}{2} \right)^2} \quad (24)$$

The coefficient of the heat conductivity according to Lehmann and Brokawy is:

$$\lambda_m = \bar{a} \sum x_i \lambda_i + (1 - \bar{a}) \frac{1}{\sum \frac{x_i}{\lambda_i}} \quad (25)$$

The specific heat under the constant pressure for the lubricating substance is:

$$c_1 = \frac{c}{\sqrt{\rho_{15}^{15}}} = \frac{1}{\sqrt{\rho_{15}^{15}}} (0,403t + 0,000405t^2) \quad (26)$$

A METHOD OF THE SOLUTION OF AN EXAMPLE

The presented mathematics model of compression process takes a form of differential equations, depicting a change of the selected parameters along the cylinder's length in the form, which is directly useful for computer programs:

The initial values used for mathematics models solution is: 1. initial air pressure equal atmospheric pressure, 2. initial specific enthalpy value of the air in the compressors' entrance.

The presented system of differential equations can be solved by numeric integration methods. The Runge-Kutty fifth range method has been adopted in order to solve the above mentioned differential equations.

The equation of state in the vapour phase for the pure components

The experiments prove that the parameters of the state of equilibrium can not change unlimited but they are interrelated by mathematics dependencies known as equations of state.

In order to verify the enthalpy calculations as based upon the mass conservation law, momentum conservation, energy conservation law, one can take an advantage of dependencies presented below.

The thermodynamic equation of state for the real gases can not be presented in a form of simple dependence which includes only one individual constant, such as the Clapeyron's formula. This is why a whole range of theoretical, empirical and half-empirical equations has been brought into to depict a state of real gases, i.e. the Kammerlingh-Onnes formula or the Dupree's formula. The real gas equations are depicted in the coefficient of compressibility:

$$\xi = \frac{pv}{RT} \quad (27)$$

The most frequently used and most precise at the same time method of confinement coefficient identification is the Kammerlingh-Onnes formula:

$$\xi = \frac{pv}{RT} = 1 + \frac{B(T)}{v} + \frac{C(T)}{v^2} + \frac{D(T)}{v^3} + \dots \quad (28)$$

$$B = x_w^2 (-0,094534 * t^2 + 20,77t - 1610,897) + 2x_w x_g (-0,0014469t^2 + 0,32391t - 42,025) + x_g^2 (-0,001329t^2 + 0,27951t - 13,183) \quad (29)$$

Specific enthalpy of wet air:

$$h_{p,T,a} = c_{pg} \int_o^t t + x c_{pw} \int_o^t t + x r_o + \int_{p_o}^{p_k} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right] dp \quad (30)$$

Specific enthalpy of vapour lubricant in higher pressure

On the enthalpy of lubricant [by B. P. Wojnowa) in vapour compose energy for heat liquid lubricant from temperature 0 [°C] to temperature of boiling, heat for evaporation and overheating vapour from temperature of boiling to temperature t [10].

In that case:

$$h_{p,T,r} = \int_0^{t_b} c_{pc} dt + r_r + \int_{t_b}^t c_{pr} dt - 4,4 \frac{p_{zr}}{T_{zr}^3} \frac{T}{M} =$$

$$= (50,2 + 0,109t + 0,00014t^2) \left(4 - \rho_{15}^{15}\right) - 73,8 - 4,4 \frac{p}{KT^2} (1,05t_{wzr} + 160 + 23)^2 \quad (31)$$

where: K – for lubricant equal 5,5.

Specific enthalpy of liquid lubricant

$$h_{p,T,e} = \int_0^t c_{pc} dt = \int_0^t \frac{1}{\sqrt{\rho_{15}^{15}}} (0,403 + 0,00081t) dt = (0,403t + 0,00081t^2) \frac{1}{\sqrt{\rho_{15}^{15}}} \quad (32)$$

$$h_{p,T,e} = \int_0^t c_{pc} dt = \int_0^t \frac{1}{\sqrt{\rho_{15}^{15}}} (0,403 + 0,00081t) dt = \gamma \frac{1}{\sqrt{\rho_{15}^{15}}} \quad (33)$$

CONCLUSIONS

Currently the program is tested. Initial calculations show differences specific enthalpy between specific enthalpy of dry air, wet air and mixture of wet air and lubricant in liquid and aerosol state in the elevated pressure and temperature.

T	Specific enthalpy of wet air ^{*)}	Specific enthalpy of lubricant ^{*)}	Specific enthalpy of mixture ^{*)}
K	kJ/kg	kJ/kg	kJ/kg
298	809,26	46,15516394	809,7264089
308	822,42	65,53847094	823,0778743
318	835,55	85,45291269	836,4038873
328	848,64	105,9199671	849,7028627
338	861,76	126,9426286	863,0291637
353	881,49	159,5379534	883,0813217

^{*)} The first results of testings, when the pressure of compressed air is 0,2 [MPa]

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